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Second Class Current in QCD Sum Rules

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Abstract

Induced tensor charge of the nucleon g_T, which originates from G-parity violation, is evaluated from QCD sum rules. We find that g_T/g_A with g_A being the axial charge is -0.0152 ± 0.0053 which is proportional to u-d quark mass difference. This result is small compared to preliminary analysis of the experiment, but is consistent with the estimate in the MIT bag model.

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I. INTRODUCTION

Deriving the coupling constants from QCD [1] is one of the most important themes in hadron physics. Also, getting precise values of the coupling constants from the first principle will enable us to make more quantitative predictions for nuclear systems.

For extracting the hadronic quantities from QCD, QCD Sum Rules (QSR) discussed below are known to be one of the two powerful tools. (The other is lattice QCD simulations [2].) QSR was first proposed in a paper by Shifman Vainsthein, and Zakharov in 1978 [3], in which the main idea and the application to meson systems such as meson masses, decay rates and $\rho-\omega$ mixing, are shown. Later, many applications have been made [4,5]. The extension to the baryon systems was put forward by Ioffe [6]. QSR with external field was also proposed by Ioffe and Smilga [7], which will be discussed later.

In this paper we will evaluate induced tensor charge of the nucleon (g_T) with the help of axial charge (g_A) and nucleon sum rules.

g_T and g_A are defined by the nucleon matrix element of the axial current,

$$\begin{aligned}&\langle P(p_2)|A^5_\mu|N(p_1)\rangle\\&=\bar{u}_p(p_2)(\gamma_\mu\gamma_5 g_A+\frac{\gamma_5 g_P}{M_p+M_n}+\frac{i\gamma_5\sigma_{\mu\nu}q^\nu}{M_p+M_n}g_T)u_n(p_1),\quad q=p_1-p_2,\end{aligned} \tag{1}$$

where $M_p(M_n)$ is proton (neutron) mass, $A^5_\mu=\bar{u}\gamma_\mu\gamma_5 d$ represents axial current and $u_p(u_n)$ reveals proton (neutron) wave function. g_P is called induced psudoscalar constant, which will not be examined in this paper. The deviation of g_A from unity is a reflection of the underlying composite structure of the nucleon and there have been many studies on g_A, e.g., QCD sum rules [8,9], quark models [10] , the bag model [11] and the skyrme model [12]. The difference of g_A and g_T is classified by G-parity which is the charge conjugation C combined with the rotation of 180° around the y-axis in the isospin space

$$G=Ce^{iI_y\pi}, \tag{2}$$

where I_y is the rotational matrix around the y-axis in the isospin space.

Under the G-parity,

$$G\bar{p}\gamma_\mu\gamma_5 nG^{-1}=-\bar{p}\gamma_\mu\gamma_5 n,\quad G\bar{p}\sigma_{\mu\nu}\gamma_5 nG^{-1}=\bar{p}\sigma_{\mu\mu}\gamma_5 n,\quad GA^5_\mu G^{-1}=-A^5_\mu. \tag{3}$$

The first current in eq.(3) with the same sign as that of A^5_μ under G- parity is called the *first class current*, while the second current in eq.(3) with the opposite sign as that of A^5_μ is referred to as the *second class current* [14].

There are two sources of G-parity violation in the standard model. One is from QED (electric charges of u and d quarks are different) and another is from the mass term in the QCD Hamiltonian (masses of u and d quarks are different). We will exclusively examine the latter effect in this thesis.

The mass term in the QCD Hamiltonian is written as

$$H_{mass}=\frac{1}{2}(m_u+m_d)(\bar{u}u+\bar{d}d)+\frac{1}{2}(m_u-m_d)(\bar{u}u-\bar{d}d). \tag{4}$$

where the light quark masses are determined from analyses of the hadron mass splittings in QCD sum rules [13];

$$m_u(\mu = 1\mathrm{GeV}) \;=\; (5.1 \pm 0.9)\mathrm{MeV},\;\; m_d(\mu = 1\mathrm{GeV}) = (9.0 \pm 1.6)\mathrm{MeV}.$$

Under G-parity, the first term in eq.(4) does not change the sign, but the second term in eq.(4) changes sign, which means that g_T is represented as $g_T \sim (m_u - m_d)/M_N$ since g_T is dimensionless. This implies that g_T will be much smaller than g_A because of the small $u-d$ quark mass difference $g_T \sim 4\mathrm{MeV}/1000\mathrm{MeV} \sim 0.004$ where we have used the quark mass difference $m_d - m_u \sim 5\mathrm{MeV}$.

This rough estimate of g_T using the G-parity violation, however, may not be consistent with the analysises of the experimental data given by measuring the beta-ray angular distribution in aligned ^{12}B and ^{12}N [15]. In Ref. [15], results of the analyses using the experimental data are quoted as

$$g_T/g_A = 0.14 \pm 0.10 \;\text{ in 1985,} \tag{5}$$
$$g_T/g_A = -0.21 \pm 0.14 \;\text{ in 1992.} \tag{6}$$

This shows that g_T is of order 10 % compared to g_A, which is order of magnitude larger than the naive expectation. Although the experimental error bars are large and even the sign of g_T/g_A is not certain yet, the data poses a theoretical challenge to give more reliable estimate of g_T/g_A.

It is in order here to show other examples of the G-parity violation which is more firmly established than g_T [16]:

1)Proton-neutron mass difference.

Experimental mass difference is $M_p - M_n = -1.29$ MeV, in which the contribution of the $u-d$ quark mass difference after subtracting the theoritical electromagnetic effect (0.76±0.3 MeV) is -2.05 MeV. This last number has been successfully reproduced in QSR calculations [17].

2)$\rho^0 - \omega$ mixing.

The $\rho - \omega$ mixing is defined by the covariant matix element $\langle \rho^0 | H_{GPB} | \omega \rangle$ at the $\rho^0 - \omega$ mass shell with H_{GPB} being the second term in eq.(4). The recent measurement of the $e^+e^- \to \pi^+\pi^-$ shows an unambigous determination of the $\rho^0 - \omega$ mixing with negative sign, which ought to be dominated by the quark mass difference since the electromagnetic effect by $\rho \to \gamma \to \omega$ is positive and small [18,19].

3)$\pi^\pm - \pi^0$ mass differnce ($m_{\pi^\pm} - m_{\pi^0}$=4.6 MeV).

This is a typical example of the electromagnetic G-parity violation. Theoretical estimate gives $(m_{\pi^\pm} - m_{\pi^0})_{em} = 4.6 \pm 0.1$ MeV, while the effect of the quark mass difference appears only in second order of the quark masses and is extremely small [20].

The ingredients of this paper are twofolds: i) to get QSR with an external field for g_T and g_A in section II-V, and ii) to predict the value of g_T relative to g_A in section VI.

As for i), we will adopt a method proposed by Ioffe and Smilga [7] and independently by Balitsky and Yung [21], in which two point functions with an external electromagnetic field strength $F_{\mu\nu}$ is studied up to linear in $F_{\mu\nu}$. The method has been applied for the magnetic moment of the nucleon and the results agree with the experimental data with a good accuracy. A method on two point functions with an axial-vector field Z_μ was also

developed by Belyaev and Kogan [8], and later improved by Pasupathy et al. [9]. They have considered terms proportional to Z_μ for evaluating the axial charge g_A. The latter method with Z_μ replaced by the vector potential A_μ is, however, not suitable for studing magnetic moments since the explicit momentum transfer must be retained.

Adopting QSR with the external field induces new parameters which are absent in ordinary QSR. These parameters reflects the response of QCD vacuum to the external field. For instance, $\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_E$, which is identical to zero in the vacuum, acquires the non-zero value due to the presence of the external field. To evaluate these new condensates, QSR with the assumption of the vector dominance can be used [22].

There is another new feature of QSR with the external field compared to the ordinary one. The phenomenological side of the correlation function with external field takes the following double pole form near the nucleon resonance

$$\langle 0|\eta|N\rangle\langle N|J^E|N\rangle\langle N|\bar{\eta}|0\rangle(p^2-M_N^2)^{-2}, \tag{7}$$

where J^E is a current coupled to the external field. Besides the double pole part which we are interested in, single poles, which expresses transition from the ground state to excited states, appears. Furthremore, the single pole term is not suppressed compared to the double pole term after applying the Borel transform. This bears no resemblance to the contribution of continuum which is exponentially suppressed by Borel transform. Hence we must take into account both the double pole and the single pole in phenomenological side on the same footing, which requires a procedure to subtract the single poles.

As for ii), one must remember that g_T originates from the G-parity violation induced by the u-d quark mass difference and the electromagnetism. The experimental value of g_T is still uncertain as we have mentioned above. Thereby we shall try to determine g_T from QSR with main emphasis on the effect of the u-d quark mass difference. Within our knowledge, no serious evaluation of g_T has been done so far except for a rough estimate using the MIT bag model [23,24]. We will therefore reexamine the bag model calculation also and compare it with our QSR result.

The paper is organized as follows. Section II-IV are devoted to derive g_T and g_A sum rules in QSR with the external field. In section V, we estimate the quark and induced condensates by using QSR. In section VI, we analyse the g_T sum rule and get its numerical number. In section VII, discussions and summary are made.

II. WEAK INTERACTION IN HADRONIC SIDE AND QCD SIDE

We start with the two point function with an external field;

$$\Pi_E(p) = i\int d^4x e^{ip\cdot x}\langle 0|T\eta_p(x)\bar{\eta}_n(0)|0\rangle_E \tag{8}$$

$$= F_{\mu\nu}\Pi_{\mu\nu}(p), \tag{9}$$

where 'E' denotes external field of weak boson W^+, $F_{\mu\nu}(x) = \partial_\mu W_\nu^+(x) - \partial_\nu W_\mu^+(x)$, and $\eta_p(\eta_n)$ corresponds to the proton (neutron) interpolating field defined as [6]

$$\eta_p(x) = \epsilon_{abc}(u^a(x)C\gamma_\mu u^b(x))\gamma_5\gamma_\mu d^c(x), \quad \eta_n(x) = \eta_p(u\leftrightarrow d) \tag{10}$$

The hadronic side of sum rules can be saturated by a process where neutron turns into proton by absorbing the W^+ boson, matrix element of which is shown in eq.(1). Hence we define an effective Lagrangian in which nucleon current couples to W^+ field as

$$L_{int}^{had} = -\frac{g}{2\sqrt{2}} j_\mu^5 W_\mu^+ = -\frac{g}{2\sqrt{2}} \bar{p} \left(g_A \gamma_\mu \gamma_5 W_\mu^+ + \frac{g_T}{M_p + M_n} \gamma_5 \sigma_{\mu\nu} \partial_\nu W_\mu^+ \right) n, \tag{11}$$

where p (n) represents proton (neutron) field and g is associated with Fermi constant as $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$ with M_W being W^+ boson mass in the Glashow-Weinberg-Salam model [25].

It is in order here to mention g_p. g_P is associated with g_A via PCAC and can be directly measured by the muon capture [24]. However, our external field does not pick up the contribution of g_P because we are using the field strength $F_{\mu\nu}$ instead of the vector potential W_μ^+ as an external field.

On the other hand, in the quark level, the interaction of quarks and the external field is written as

$$L_{int}^{quark} = \frac{-g}{2\sqrt{2}} j_\mu^5 W^+ = \frac{-g}{2\sqrt{2}} \bar{u} \gamma_\mu \gamma_5 d \ \ W_\mu^+, \tag{12}$$

where u and d are up and down quark respectively. The common factor $g/2\sqrt{2}$ in eq.(11) and in eq.(12) is obtained by comparing the V-A theory [26] with the Glashow-Weinberg-Salam model.

III. HADRONIC SIDE FOR TWO POINT FUNCTION WITH EXTERNAL FIELD

In this section we examine the hadronic contribution to QSR with the external field. Firstly, we consider Fig.1, which shows that a neutron with momentum p_1 absorbs W^+ boson and turns into proton with momentum p_2. Using eq.(11), we may write down Fig.1 as

$$\text{Fig.1} = \frac{1}{\hat{p}_2 - M_p} \left(g_A \gamma_\mu \gamma_5 + \frac{i\gamma_5 \sigma_{\mu\nu}}{M_p + M_n} g_T q_\nu \right) \frac{1}{\hat{p}_1 - M_n} \tag{13}$$

$$= \frac{1}{(p_2^2 - M_p^2)(p_1^2 - M_n^2)} \times \left(-\frac{\hat{q}}{2} + \hat{p} + M_p \right) \left(g_A \gamma_\mu \gamma_5 + \frac{i\gamma_5 \sigma_{\mu\nu}}{M_p + M_n} g_T q_\nu \right) \left(\frac{\hat{q}}{2} + \hat{p} + M_n \right), \tag{14}$$

where $p_2 (p_1)$ is proton (neutron) momentum mentioned above, $p = \frac{p_1 + p_2}{2}$ and $q = p_1 - p_2$. In the limit of soft external momentum $q_\mu \to 0$, we keep only the terms proportional to q_μ in eq.(14) to extract $F_{\mu\nu}$. Then eq.(14) is reduced to

$$\frac{i\gamma_5 q_\nu}{(p^2 - M_n^2)(p^2 - M_p^2)} \left[P_1 \hat{p} \sigma_{\mu\nu} + P_2 \sigma_{\mu\nu} \hat{p} + P_3 \sigma_{\mu\nu} + P_4 (\gamma_\mu p_\nu - \gamma_\nu p_\mu) \hat{p} \right] \equiv q_\nu \Gamma_{\mu\nu}(p), \tag{15}$$

where

$$P_1 = -\frac{g_A}{2} - \frac{M_n}{M_n + M_p} g_T, \tag{16}$$

$$P_2 = -\frac{g_A}{2} + \frac{M_p}{M_n + M_p} g_T, \tag{17}$$

$$P_3 = -\frac{g_A}{2}(M_n - M_p) + (\frac{M_n M_p - p^2}{M_p + M_n}) g_T, \tag{18}$$

$$P_4 = \frac{2ig_T}{M_p + M_n}. \tag{19}$$

By using eq.(11), we evaluate eq.(8) in first order of the external field:

$$\begin{aligned}
& - \int d^4x e^{ip\cdot x} \langle 0 | T(\eta_p(x) \bar{\eta}_n(0) L^{had}_{int}) | 0 \rangle \\
&= -\frac{g}{2\sqrt{2}} \lambda_p \lambda_n \int d^4y \frac{1}{(2\pi)^4} \int d^4l e^{i(p-l)\cdot y} W^+(y) \\
&\times \frac{1}{\hat{p} - M_p} \left(g_A \gamma_\mu \gamma_5 + \frac{i\gamma_5 \sigma_{\mu\nu}}{M_p + M_n} g_T (l-p)_\nu \right) \frac{1}{\hat{l} - M_n} \\
&= -\frac{g}{2\sqrt{2}} \lambda_p \lambda_n \frac{1}{(2\pi)^4} \int d^4y \int d^4q e^{-iq\cdot y} q^\nu \Gamma_{\mu\nu}(p) W^+(y) \\
&= -\frac{ig}{4\sqrt{2}} \lambda_p \lambda_n \Gamma_{\mu\nu} F_{\mu\nu}(0) \\
&= \frac{g\gamma_5 \lambda_n \lambda_p}{4\sqrt{2}(p^2 - M_n^2)(p^2 - M_p^2)} F_{\mu\nu}(0) \\
&\times \left[P_1\ \hat{p}\sigma_{\mu\nu} + P_2\ \sigma_{\mu\nu}\hat{p} + P_3\ \sigma_{\mu\nu} + P_4\ (\gamma_\mu p_\nu - \gamma_\nu p_\mu)\hat{p} \right],
\end{aligned} \tag{20}$$

where P_1, P_2, P_3 and P_4 are defined in eq.(16)-eq.(19), λ_n and λ_p are defined as $\langle 0|\eta|N\rangle = \lambda_N u(p)$ with $u(p)$ being the nucleon Dirac spinor. Apart from the terms above, we must take into account two other contributions. One is the single pole caused by a transition of nucleon to resonance states as follows,

$$\Pi_E(p) \sim \lambda_N \lambda_{N^*} \frac{1}{\hat{p} - M_N} H_{NN^*} \frac{1}{\hat{p} - M_{N^*}}, \tag{21}$$

where $N(N^*)$ is the nucleon (excited states, e.g., $N(1440)$), and H_{NN^*} is a transition matrix from the nucleon to the excited states. As we have mentioned, the single pole is not suppressed compared to the double pole after the Borel transform. Since we are not interested in the single poles, we will subtract them, using a procedure shown later. The other hadronic contribution is a continuum starting at a threshold S_0, which contains only the excited states. The continuum can be exponentially suppressed by applying the Borel transform.

IV. OPE FOR TWO POINT FUNCTION WITH EXTERNAL FIELD

In ordinary QSR with no external fields, only Lorenz invariant operators survive in OPE. On the other hand, the external field induces new condensates. Relevant condensates up to dimension 6 in our case read

$$\langle 0|\bar{u}\gamma_5\sigma_{\mu\nu}d|0\rangle_E = (m_u - m_d)F_{\mu\nu}\frac{g}{2\sqrt{2}}\chi(0), \tag{22}$$

$$g_s\langle 0|\bar{u}\gamma_5 G_{\mu\nu}d|0\rangle_E = \langle\bar{d}d - \bar{u}u\rangle_0 F_{\mu\nu}\frac{g}{2\sqrt{2}}\kappa(0), \tag{23}$$

$$g_s\epsilon_{\mu\nu\rho\omega}\langle 0|\bar{d}G_{\rho\omega}u|0\rangle_E, = \langle\bar{d}d - \bar{u}u\rangle_0 iF_{\mu\nu}\frac{g}{2\sqrt{2}}\xi(0). \tag{24}$$

The above condensates are non-vanishing because the QCD vacuum is distorted by the external field and the Lorenz invariance is broken. Note also that taking $m_u = m_d$ makes all the above terms vanish. Using the fixed point gauge, we can rewrite the $i\langle 0|T(\eta_p(x)\bar{\eta}_n(0))|0\rangle_E$ as follows:

$$i\langle 0|\ T(\eta_p(x)\bar{\eta}_n(0))\ |0\rangle_E = 4i\epsilon_{abc}\epsilon_{def}\langle 0|\ \gamma_5\gamma_\mu iS_d^{ce}(x)\gamma_\nu CiE_{ad}^T(x)C\gamma_\mu iS_u^{bf}\gamma_\nu\gamma_5\ |0\rangle_E \tag{25}$$

with

$$\begin{aligned}
&iS_q^{ab}(x) = \langle 0|T(q^a(x)\bar{q}^b(0))|0\rangle \\
&= \frac{i\hat{x}}{2\pi^2x^4}\delta^{ab} + \frac{ix_\alpha}{8\pi^2x^2}(t^e)^{ab}\tilde{G}_e^{\alpha\rho}\gamma_\rho\gamma_5 - \frac{m\delta^{ab}}{4\pi^2x^2} + \langle\chi_q^a(x)\bar{\chi}_q^b(0)\rangle_0, && (26)\\
&iE^{ab}(x) = \langle 0|T(u^a(x)\bar{d}^b(0)|0\rangle_E \\
&= \frac{ig}{2\sqrt{2}}\delta^{ab}\frac{x_\lambda}{8\pi^2x^2}\tilde{F}_{\lambda l}\gamma_l + \langle\chi_u^a(x)\bar{\chi}_d^b(0)\rangle_E && (27)\\
&-(m_u - m_d)\left\{\frac{1}{32\pi^2}(\log(-x^2\Lambda^2/4) + 2\gamma_E)\gamma_5\sigma_{\rho\nu}F^{\rho\nu} + \frac{i}{16\pi^6x^2}F_{\rho\mu}\gamma_5\gamma_\mu\hat{x}x_\rho\right\},
\end{aligned}$$

where $iE^{ab}(x)$ is calculated by eq.(12) with the first order perturbation, $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\omega}F^{\rho\omega}$, and Λ^2 is an infrared cut off parameter.

$\langle\chi_u^a(x)\bar{\chi}_d^b(0)\rangle_E$ in eq.(27) expresses non-perturbative condensate under the external field, while other terms are coupled directly to the quark propagator. To calculate the nonperturbative terms, we shall compare it with $\langle\chi_q^a(x)\bar{\chi}_q(0)\rangle_0$ which is expanded as

$$\langle\chi^a(x)\bar{\chi}^b(0)\rangle_0 = \langle 0|q^a(x)\bar{q}^b(0)|0\rangle \tag{28}$$

$$= -\frac{\delta^{ab}}{12}\langle 0|\bar{q}q|0\rangle - \frac{\delta^{ab}x^2}{192}\langle 0|g_s\bar{q}\sigma\cdot Gq|0\rangle + \cdots \tag{29}$$

In the case above, we have retained only the Lorentz scalar operators. In contrast, $\langle\chi_u^a(x)\chi_d^b(0)\rangle_E$ is expanded only by the Lorenz tensor terms corresponding to the induced condensates eq.(22)$-$ eq.(24). Hence

$$\begin{aligned}
\langle\chi_u^a(x)\bar{\chi}_d^b(0)\rangle_E &= -\frac{1}{24}\delta_{ab}\gamma_5\sigma_{\mu\nu}\langle\bar{u}\gamma_5\sigma_{\mu\nu}d\rangle_E - \frac{x_\rho x_\omega}{48}\gamma_5\sigma_{\mu\nu}\langle\bar{u}\gamma_5\sigma_{\mu\nu}D_\rho D_\omega d\rangle_E + \cdots \\
&= -\frac{1}{24}\frac{g}{2\sqrt{2}}\delta^{ab}\gamma_5\sigma_{\mu\nu}F^{\mu\nu}(m_u - m_d)\chi(0) \\
&\quad -\frac{g}{2\sqrt{2}}\frac{1}{3^2 2^5}\gamma_5\sigma_{\mu\nu}\langle\bar{d}d - \bar{u}u\rangle_0 \\
&\quad \times\Big\{(\kappa(0) - \xi(0))x^2F_{\mu\nu} - (2\kappa(0) + \xi(0))x_\mu x_\omega F_{\nu\omega}\Big\} + \cdots
\end{aligned} \tag{30}$$

We turn to carry out OPE for eq.(25) with diagrams Fig. 2(a)-(j). For the chiral odd structures, we impose $m_u = m_d$. This induces $P_1 = P_2 = -g_A/2$ in eq.(20) due to $g_T = 0$. The chiral odd structure can be applied to estimate the axial-charge (g_A) with $m_u = m_d$.

On the other hand, the chiral even structures in eq.(25) are evaluated up to linear in $(m_u - m_d)$, which leads to g_T sum rule.

Let us examine each contribution in Fig.2 more closely
The coefficient of $F_{\mu\nu}$ with the chiral odd structure given in Fig.2(a) reads

$$\text{Fig.2(a)} = -\frac{g}{2\sqrt{2}}\frac{6}{\pi^6 x^8}x_\lambda \gamma_l \tilde{F}_{\lambda l}. \tag{31}$$

The coefficient of $F_{\mu\nu}\langle\frac{\alpha_s}{\pi}G^2\rangle$ with the chiral odd structure given in Fig.12(b) reads

$$\text{Fig.2(b)} = -\frac{g}{2\sqrt{2}}\frac{1}{32\pi^4 x^4}x_\lambda \tilde{F}_{\lambda l}\gamma_l \left\langle \frac{\alpha_s}{\pi}G^2 \right\rangle_0, \tag{32}$$

where $\left\langle \frac{\alpha_s}{\pi}G^2 \right\rangle_0 = 0.012(\text{GeV}^2)$ is called gluon condensate, whose value is determined from the analysis of heavy quark system based on QSR. The coefficient of $F_{\mu\nu}\langle\bar{q}q\rangle^2$ with the chiral odd structure given in Fig.12(c) reads

$$\text{Fig.2(c)} = -\frac{g}{2\sqrt{2}}\frac{\langle\bar{q}q\rangle_0^2}{12\pi^2 x^2}x_\lambda \tilde{F}_{\lambda l}\gamma_l. \tag{33}$$

The coefficient of $(m_u - m_d)F_{\mu\nu}$ coupled directly to the propagator given in Fig.2(d) reads

$$\text{Fig.2(d)} = -i\frac{g}{2\sqrt{2}}\frac{3}{2\pi^6 x^8}x_\lambda \tilde{F}_{\lambda l}\gamma_l \hat{x}(m_u - m_d). \tag{34}$$

Fig.2(f) also gives a coefficient of $(m_u - m_d)F_{\mu\nu}$. However, it causes the infrared divergence which must be absorbed into the same dimensional operator shown in Fig.2(e).
The coefficient of $\langle\bar{d}\gamma_5\sigma_{\mu\nu}u\rangle_E$, which is dimension 3 and is given in Fig.2(e), reads

$$\text{Fig.2(e)} = i\frac{1}{\pi^4 x^8}\gamma_5 \hat{x}\sigma_{\rho\omega}\hat{x}\langle\bar{d}\gamma_5\sigma_{\rho\omega}u\rangle_E. \tag{35}$$

The coefficient of $\langle\bar{d}d - \bar{u}u\rangle_0 F_{\mu\nu}$ given in Fig.2(g) reads

$$\text{Fig.2(g)} = -i\frac{g}{2\sqrt{2}}\frac{1}{2\pi^4 x^6}x_\lambda \tilde{F}_{\lambda l}\hat{x}\gamma_l \langle\bar{d}d - \bar{u}u\rangle_0. \tag{36}$$

The coefficient of $g_s\langle\bar{d}\gamma_5 G_{\mu\nu}u\rangle_E$ and $g_s\epsilon^{\mu\nu\rho\omega}\langle\bar{d}G_{\rho\omega}u\rangle_E$ given in Fig.12(h) and (i) read

$$\text{Fig.2(h) and (i)} = i\frac{\gamma_5\sigma_{\rho\omega}}{12\pi^4 x^4}[g_s\langle\bar{d}\gamma_5 G^{\rho\omega}u\rangle_E + ig_s\epsilon^{\rho\omega\mu\nu}\langle\bar{d}G_{\mu\nu}u\rangle_E] \tag{37}$$

$$+\frac{\gamma_5(\gamma_\rho x_\omega - \gamma_\omega x_\rho)\hat{x}}{4\pi^4 x^6}[g_s\langle\bar{d}\gamma_5 G^{\rho\omega}u\rangle_E + \frac{1}{2}g_s i\epsilon^{\rho\omega\mu\nu}\langle\bar{d}G_{\mu\nu}u\rangle_E], \tag{38}$$

where the contribution of Fig.2(h) is zero bacause $\langle 0|\bar{d}u|0\rangle = 0$ and in Fig.2(i) the gluon field emitted by a soft quark interacts W^+ field via quark condensate.
The coefficient of $g_s\langle\bar{d}\gamma_5 G_{\mu\nu}u\rangle_E$ and $g_s\epsilon^{\mu\nu\rho\omega}\langle\bar{d}G_{\rho\omega}u\rangle_E$ given in Fig.2(j) read

$$\text{Fig.2(j)} = -i\frac{\gamma_5(\gamma_\rho x_\omega - \gamma_\omega x_\rho)\hat{x}}{8\pi^4 x^4}\hat{x}[g_s\epsilon^{\rho\omega\mu\nu}\langle\bar{d}G_{\mu\nu}u\rangle_E - 2ig_s\langle\bar{d}\gamma_5 G^{\rho\omega}u\rangle_E] \tag{39}$$

$$-i\frac{\gamma_5\sigma_{\rho\omega}}{4\pi^4 x^4}g_s\langle\bar{d}\gamma_5 G^{\rho\omega}u\rangle_E, \tag{40}$$

where the gluon field emitted by a hard quark interacts W^+ field via quark condensate.

In summary, we obtain the following formula

$$\Pi_E(p) = \frac{g}{2\sqrt{2}}F_{\mu\nu}\left[Q_1\gamma_5(\hat{p}\sigma_{\mu\nu} + \sigma_{\mu\nu}\hat{p}) + \{Q_2\gamma_5\sigma_{\mu\nu} + Q_3 i\gamma_5(\gamma_\mu p_\nu - \gamma_\nu p_\nu)\hat{p}\}(m_u - m_d)\right], \tag{41}$$

$$Q_1 = \frac{-1}{32\pi^4}p^2\log(-p^2) - \frac{1}{64\pi^2}\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{p^2} - \frac{\langle\bar{q}q\rangle_0^2}{6p^4}, \tag{42}$$

$$Q_2 = -p^2\log(-p^2)\left(\frac{1}{16\pi^4} - \frac{\chi(0)}{24\pi^2}\right) - C_m\left(\frac{1}{8\pi^2} - \frac{\kappa(0)}{6\pi^2} - \frac{\xi(0)}{12\pi^2}\right)\log(-p^2), \tag{43}$$

$$Q_3 = \log(-p^2)\left(\frac{1}{32\pi^4} - \frac{\chi(0)}{12\pi^2}\right) + \frac{C_m}{8\pi^2 p^2}, \tag{44}$$

where we have used a relation $\langle\bar{d}d - \bar{u}u\rangle_0 = C_m(m_u - m_d)$ which will be discussed below.

V. THE ESTIMATE OF THE QUARK AND INDUCED CONDENSATES

Before setting up the sum rules, we need to estimate the magnitude of the quark and induced condensates. For the quark condensate, we utilize Finite Energy Sum Rules (FESR) [27,28] for nucleon mass, where we look for the optimal quark condensate which reproduce the nuclon mass within the standard values of the condensate $\langle\bar{q}q\rangle_0(1\text{GeV}^2) = -(225 \pm 25\text{MeV})^3$.

From ref. [28], we get the FESR for nucleon as follows:

$$64\pi^4\lambda_N^2 = \frac{S_N^3}{3} + 2\pi^2\langle\frac{\alpha_s}{\pi}G^2\rangle_0 S_N + \frac{128}{3}\pi^4\langle\bar{q}q\rangle_0^2, \tag{45}$$

$$64\pi^4\lambda_N^2 M_N = -8\pi^2\langle\bar{q}q\rangle S_\pi^2 + \frac{32}{9}\pi^4\langle\bar{q}q\rangle_0\langle\frac{\alpha_s}{\pi}G_0^2\rangle_0, \tag{46}$$

$$64\pi^4\lambda_N^2 M_N^2 = \frac{S_N^4}{4} + \pi^2\langle\frac{\alpha_s}{\pi}G^2\rangle_0 S_N^2 - \frac{128}{9}\pi^4\langle\bar{q}q\rangle_0^2\frac{\alpha_s}{\pi}S_N, \tag{47}$$

where λ_N is defined above, and S_N is the continuum threshold of nucleon sum rules. Solving eq.(45)-(47) numerically, we get results in Table 1. Hence we will utilize the following numbers in the anaylses of g_T sum rule later;

$$\langle\bar{q}q\rangle(\text{GeV}^3) = (-0.2185)^3, \quad S_N(\text{GeV}^2) = 1.6, \quad \lambda_N(\text{GeV}^3) = 0.0188. \tag{48}$$

$\langle\bar{q}q\rangle_0$	$(-0.250\text{GeV})^3$	$(-0.230\text{GeV})^3$	$(-0.2185\text{GeV})^3$	$(-0.210\text{GeV})^3$
$S_N(\text{GeV}^2)$	2.27	1.84	1.60	1.42
$\lambda_N(\text{GeV}^3)$	0.0296	0.0224	0.0188	0.0162
$M_N(\text{GeV})$	1.15	1.024	0.940	0.872

Table 1: S_N, λ_N, M_N obtained from Eq.(45) $\sim$ (47) with four different values of $\langle \bar{q}q \rangle_0$.

Now we turn to the calculation of induced condensates with the help of QSR.
We first expand eq.(22) in terms of W^+ up to first order.

$$\langle \bar{d}\gamma_5\sigma_{\mu\nu}u\rangle_E = -i\frac{g}{2\sqrt{2}}\int d^4x\langle 0|T(\bar{d}\gamma_5\sigma_{\mu\nu}u(0)\bar{u}\gamma_\rho\gamma_5 d(x))|0\rangle\ W^+(x) \tag{49}$$

$$= -i\frac{g}{2\sqrt{2}}\int d^4x\ \Pi_{\mu\nu,\rho}(x)W^+_\rho(x),$$

where $$\Pi_{\mu\nu,\rho}(x) = \langle 0|T(\bar{d}\gamma_5\sigma_{\mu\nu}u(0)\bar{u}\gamma_\rho\gamma_5 d(x))|0\rangle. \tag{50}$$

To estimate $\Pi_{\mu\nu,\rho}(x)$, we expand eq. (50) in terms of the local operators up to dimension 5, whose diagrams are Fig.3(a)-(c), and retain the terms proportional to quark mass. Then we get the following equation:

$$\Pi_{\mu\nu,\rho}(q) = (-q_\mu g_{\rho\nu} + q_\nu g_{\rho\mu})(m_u - m_d)\chi(q^2) \tag{51}$$

with

$$\chi(q^2) = \frac{3}{8\pi^2}\log(-q^2) + \left(\frac{1}{q^2} + \frac{m_0^2}{3q^4}\right)C_m, \tag{52}$$

where the first, second and third term on the right hand side correspond to Fig.3(a), (b) and (c), respectively. $m_0^2 = 0.8(\mathrm{GeV}^2)$ is defined by $\langle 0|g_s\bar{q}\sigma\cdot Gq|0\rangle = m_0^2\langle 0|\bar{q}q|0\rangle$ [30]. Using eq.(50) and eq.(51), we reach the result defined in eq.(22):

$$\langle \bar{d}\gamma_5\sigma_{\mu\nu}u\rangle_E = \frac{g}{2\sqrt{2}}F_{\mu\nu}(0)(m_u - m_d)\chi(0), \tag{53}$$

where we replace $F_{\mu\nu}(x)$ by $F_{\mu\nu}(0)$ bacause the field strength is assumed to be constant. For the phenomenological side in eq.(52), we assume that $\chi(q^2)$ is saturated by a_1 meson with mass 1260(MeV), which is the lowest state coupled to both pseudovector and pseudotensor states, and the continuum starting at S_χ:

$$\frac{1}{\pi}\mathrm{Im}\ \chi(s) = f_\chi\delta(s - m_{a_1}^2) - \frac{3}{8\pi^2}\theta(s - S_\chi). \tag{54}$$

Then, we get

$$\chi(0) = \frac{f_\chi}{m_{a_1}^2} - \frac{3}{8\pi^2}\int_0^{\Lambda^2}\frac{\theta(s - S_\chi)}{s - m_{a_1}^2}ds, \tag{55}$$

where $\Lambda^2 = 1\mathrm{GeV}^2$ is taken as a characteristic scale of seperating the perturbative and the non-perturbative part in $\langle \bar{d}\gamma_5\sigma_{\mu\nu}u\rangle_E$. Matching eq.(52) and eq.(54) using FESR, we get two sum rules,

$$n = 0, \quad \frac{3}{8\pi^2}S_\chi + C_m = f_\chi, \tag{56}$$

$$n = 1, \quad \frac{3}{16\pi^2}S_\chi^2 + \frac{m_0^2}{3} = f_\chi m_{a_1}^2. \tag{57}$$

f_χ is rewritten as

$$f_\chi = C_m + \frac{3}{8\pi^2}m_{a_1}^2 \pm \sqrt{\frac{C_m}{4\pi^2}(3M_{a_1}^2 - m_0^2) + \frac{9m_{a_1}^4}{64\pi^4}}. \tag{58}$$

To obtain f_χ, we take $C_m(\mathrm{GeV}^2) = (-0.0307) - (-0.0223)$ which makes the proton-neutron mass difference within the interval $1.95\mathrm{GeV} \leq (M_n - M_p) \leq 2.41\mathrm{GeV}$ where the electromagnetic effect are subtracted out [29]. Thus we get

$$\chi(0) = (-0.0337) - (-0.0470) \tag{59}$$

As for $g_s\langle \bar{d}\gamma_5 G_{\mu\nu} u\rangle_E$ and $g_s\langle \bar{d}\epsilon_{\mu\nu\rho\omega}G^{\rho\omega}u\rangle_E$, we make OPE up to dimension 7 and get the following results:

$$g_s\langle \bar{d}\gamma_5 G_{\mu\nu} u\rangle_E = \frac{g}{2\sqrt{2}}C_m(m_u - m_d)F_{\mu\nu}\kappa(0), \tag{60}$$

$$\text{with} \quad \kappa(q^2) = \frac{m_0^2}{12}\frac{1}{q^2} + \frac{\pi^2}{36q^4}\langle\frac{\alpha_s}{\pi}G^2\rangle_0, \tag{61}$$

$$g_s\langle \bar{d}\epsilon_{\mu\nu\rho\omega}G^{\rho\omega}u\rangle_E = i\frac{g}{2\sqrt{2}}C_m(m_u - m_d)F_{\mu\nu}\xi(0), \tag{62}$$

$$\text{with} \quad \xi(q^2) = -\frac{m_0^2}{6q^2} + \frac{1}{18q^4}\pi^2\langle\frac{\alpha_s}{\pi}G^2\rangle_0, \tag{63}$$

where graphs for OPE are shown in Fig.4. For phenomenological part, we adopt two pole approximation namely,

$$\kappa(q^2) = \frac{f_{a_1}}{m_{a_1}^2 - q^2} + \frac{f_{a_2}}{m_{a_2}^2 - q^2}, \tag{64}$$

where m_{a_1} is a_1 meson mass appeared above, m_{a_2} is a_2 meson mass (1360MeV) which is the tensor meson, and f_{a_1} and f_{a_2} represent the pole residues. The same approximation is also adopted for $\xi(q)$. Equating the OPE side to the phenomenological side and comparing the coefficients up to q^4 to determine $f_{a_1}$and f_{a_2}, we get $\kappa(0) = -0.079,\ \xi(0) = 0.163$.

VI. ANALYSIS AND NUMERICAL RESULT

From eq.(41) and (20), we have found that there exist four sum rules corresponding to the tensor structures $\hat{p}\sigma_{\mu\nu}$, $\sigma_{\mu\nu}\hat{p}$, $\sigma_{\mu\nu}$, and $(\gamma_\mu p_\nu - \gamma_\nu p_\mu)\hat{p}$. We take only two sum rules among them to evalaute g_A and g_T.

As mentioned above, sum rule for g_A is obtained from the chiral odd structure in the chiral limit. On the other hand, sum rule for g_T can be deduced from the part proportional to $(\gamma_\mu p_\nu - \gamma_\nu p_\mu)\hat{p}$ since P_4 in eq.(20) does not contain g_A. The sum rule obtained from the tensor structure $\sigma_{\mu\nu}$ is not suitable, since in the phenomenological side the contribtion of the term with g_A is comparable to that of the term with g_T, and both terms vanish in the chiral limit.

Matching eq.(20) and eq.(41) and making Borel transform, we obtain g_A and g_T sum rules in Borel sum rules (BSR) [3] as

$$\left(\frac{g_A}{M^2}+A_{sp}\right) = -\frac{e^{\frac{M_N^2}{M^2}}}{\lambda_p\lambda_n}\left[\frac{1}{8\pi^4}M^4E_1(\frac{S_A}{M^2})-\frac{1}{16\pi^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_0-\frac{2}{3}\frac{\langle\bar{q}q\rangle_0^2}{M^2}\right], \tag{65}$$

$$\left(\frac{1}{M^2}\frac{g_T}{M_p+M_n}+T_{sp}\right) = -\frac{(m_u-m_d)}{\lambda_p\lambda_n}M^2e^{\frac{M_N^2}{M^2}}\left[E_0(\frac{S_T}{M^2})\left\{\frac{1}{32\pi^4}-\frac{\chi(0)}{12\pi^2}\right\}+\frac{C_m}{8\pi^2M^2}\right], \tag{66}$$

where $E_n(x) = 1-(1+x+\frac{x^2}{2!}+\cdots+\frac{x^n}{n!})e^{-x}$, A_{sp} and T_{sp} represent the contribution of single poles coming from the nucleon - the resonance transition discussed above. To subtract A_{sp} and T_{sp}, we multiply the operator $\frac{\partial}{\partial(1/M^2)}$ to both sides of eq.(65) and (66). Assuming that A_{sp} and B_{sp} are independent of the Borel mass M^2, we obtain the final sum rules

$$g_A = \frac{e^{\frac{M_N^2}{M^2}}}{\lambda_N^2}\frac{M^6}{4\pi^4} \tag{67}$$

$$\times\left[E_2(\frac{S_A}{M^2})-\frac{M_N^2}{2M^2}E_1(\frac{S_A}{M^2})-\frac{\pi^2M_N^2}{4M^6}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_0+\frac{8\pi^4\langle\bar{q}q\rangle_0^2}{3M^6}\left\{1+\frac{M_N^2}{M^2}\right\}\right],$$

$$\frac{g_T}{2M_N} = -\frac{(m_u-m_d)}{\lambda_N^2}M^2e^{\frac{M_N^2}{M^2}}$$

$$\times\left[\left\{M_N^2E_0(\frac{S_T}{M^2})-M^2E_1(\frac{S_T}{M^2})\right\}\left(\frac{1}{32\pi^4}-\frac{\chi(0)}{12\pi^2}\right)+\frac{C_m}{8\pi^2}\frac{M_N^2}{M^2}\right], \tag{68}$$

where $S_A(S_T)$ is the threshold for $g_A(g_T)$, and $\lambda_n=\lambda_p\equiv\lambda_N$ and $M_n=M_p\equiv M_N$ are taken since m_u-m_d is extracted out in (66).

Here we make analyses of g_T sum rule. I) We make an Borel analysis using nucleon sum rules according to the procedures shown in ref. [3], and apply the FESR to get the qualitative understanding. II) We make a Borel analysis on the ratio g_T/g_A which is directly related to the experimental data, using g_A sum rule.

I) First of all, we write down two nucleon sum rules [6] in order to get rid of the coefficint $e^{M_N^2/M^2}/\lambda_N^2$ in eq.(68):

$$4\pi^4\lambda_N^2e^{-M_N^2/M^2} = \frac{M^6}{8}E_2(x_N)+\frac{\pi^2M^2}{8}\langle\frac{\alpha_s}{\pi}G^2\rangle_0E_0(x_N)+\frac{8\pi^4}{3}\langle\bar{u}u\rangle^2, \tag{69}$$

$$4\pi^4\lambda_N^2M_Ne^{-M_N^2/M^2} = -\pi^2\langle\bar{d}d\rangle M^4E_1(x_N)+\frac{2\pi^4}{9}\langle\bar{d}d\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle_0, \tag{70}$$

where $x_N=S_N/M^2$. We call eq. (69) (eq.(70)) even (odd) sume rule since it contains only even (odd) dimensional operators.

Because the formulas obtained from BSR depend on the unphysical parameter, i.e., the Borel mass M^2, we must adopt a Borel window, $M^2_{min.}<M^2<M^2_{max}$, in which g_T is independent of M^2 within this range. In other words, the threshold S_T is chosen to make the Borel curve as flat as possible in the Borel window. To obtain the Borel window, we take $1-E_0(S_T/M^2)\geq 30\%$ at $S_T=2.0$ in eq.(66) as the upper limit and get $M^2_{max}=1.66$. However, we can not get the suitable minimum in Borel window since the contribution of

the condensate term in eq.(68) is the same magnitude as that of the perturbative term. The multiplication of the operator for subtracting the single pole makes the contribution of the perturbative term reduced because of $\frac{\partial}{\partial(1/M^2)}M^2 e^{M_N^2/M^2} = 0$ at $M^2 = M_N^2$. Thus, we utilize the lower limit of the nucleon sum rule (69) where the second and third terms is less than 30 % compared to the pertubative term. Thus we arrives at a Borel window $1.22\mathrm{GeV}^2 \leq M^2 < 1.66\mathrm{GeV}^2$, and we carry out the Borel analysis on g_T with $C_m = -0.0337$ and -0.0223 to search the optimal threshold in the Borel window.

The results are summarized in Table 2 and Borel curves with the optimal threshold are shown in Fig.5 (a) and (d), which show that the magnitude of g_T in our analysis is smaller than that of the preliminary experimental value by order of magnitude. To get the physical interpretation of the result, we make FESR for g_T and get the result

$$\frac{g_T}{2M_N} = \frac{(m_u - m_d)}{\lambda_N^2}\left[\left(\frac{1}{2}S_T^2 - M_N^2 S_T\right)\left(\frac{1}{32\pi^4} - \frac{\chi(0)}{12\pi^2}\right) - \frac{C_m}{8\pi^2}M_N^2\right]. \tag{71}$$

This implies that adopting $S_T = 2M_N^2 \sim 2.5M_N^2$ as shown in Table 4 makes the first term with the threshold S_T small compared to the second term. Neglecting the first term and utilizing $\lambda_N^2 = 4\langle\bar{q}q\rangle_0^2$ and $M_N = \left(-\frac{25\pi^2}{2}\langle\bar{q}q\rangle_0\right)^{\frac{1}{3}}$ obtained by FESR, we arrive at

$$g_T = \frac{25}{32}\frac{\langle\bar{d}d - \bar{u}u\rangle_0}{\langle\bar{u}u\rangle_0} = \frac{25}{32}\frac{C_m(m_u - m_d)}{\langle\bar{u}u\rangle_0}, \tag{72}$$

which gives $g_T = (-0.00896) - (-0.00651)$ when $-0.0307 \leq C_m \leq -0.0223$ is used. This represents a good agreement with the result from BSR.

	$C_m = -0.0307$	$C_m = -0.0223$	$C_m = -0.0307$	$C_m = -0.0223$
$\langle\bar{q}q\rangle_0 \quad (S_N)$	$g_T^{even} \quad (S_T^{even})$		$g_T^{odd} \quad (S_T^{odd})$	
$(-0.2185)^3$ (1.60)	-0.0106 (2.15)	-0.00413 (1.74)	-0.0163 (2.62)	-0.00719 (2.00)

Table 2: $g_T^{even}(g_T^{odd})$ and its threshold $S_T^{even}(S_T^{odd})$ using even (odd) nucleon sum rule with two different value of C_m where $\langle\bar{q}q\rangle_0$ is in GeV^3 unit, and the thresholds S_N, S_T are in GeV^2 unit. $\langle\bar{q}q\rangle_0 = (-0.2185GeV)^3$ reproduces nucleon mass.

II) As it is customary to take the ratio of g_T and g_A, we make Borel analysis on g_T/g_A by taking the ratio of eq.(67) and eq.(68). For the threshold S_A, we take S_A which satisfies the experimental number of g_A(=1.25) in FESR, and get $S_A = 1.68\mathrm{GeV}^2$. Then FESR for g_A reads

$$g_A = \frac{1}{4\pi^4}\frac{1}{\lambda_N^2}\left[\frac{1}{6}S_A^3 - \frac{M_N^2}{4}S_A^2 - \frac{\pi^2 M_N^2}{4}\langle\frac{\alpha_s}{\pi}G^2\rangle + \frac{8\pi^4}{3}\langle\bar{q}q\rangle^2\right], \tag{73}$$

where we have used λ_N and $\langle\bar{q}q\rangle$ in eq. (48). After searching optimal threshold in the above window, we get the results in Table 3. Corresponding Borel curves are given in Fig. 6 where we use $\langle\bar{q}q\rangle_0(\mathrm{GeV}^2) = (-0.2185)^3$ to compare the results with those in case I). Note that replacing $M_{min}^2 = 1.22$ by $M_{min}^2 = 0.61$ which is obtained from g_A sum rule has

the same qualitative results with the window used in case I) and the quantitative change is within 15%.

Table 3 shows that

$$g_T/g_A = -0.0152 \pm 0.0053 \tag{74}$$

which will be the one to be compared with experimental value.

	$C_m = -0.0307$	$C_m = -0.0223$
$\langle\bar{q}q\rangle_0 \ \ (S_A)$	$g_T \ \ (S_T)$	
$(-0.2185)^3(1.68)$	-0.0205 (2.97)	-0.00983 (2.22)

Table 3: g_T/g_A and its threshold S_T with two different values of C_m where $\langle\bar{q}q\rangle_0$ is in GeV^3 unit, the thresholds S_T, S_A are in GeV^2 unit and C_m is in GeV^2 unit. $\langle\bar{q}q\rangle_0 = (-0.2185)^3$ reproduces nucleon mass.

Here we mention the uncertainty of our results originating from C_m. $\chi(0)$ grows as C_m becomes small, which changes the sign of g_T from negative to positive. Hence determining g_T in QSR does not become quite accurate unless $\langle\bar{d}d-\bar{u}u\rangle_0$ is precisely determined. Also g_A and g_T are rather sensitive to $\langle\bar{q}q\rangle_0$, thus we need to know its accurate value.

VII. DISCUSSIONS AND SUMMARY

So far, we have calculated g_T in QSR and gotten Table 2 and 3, and we found that
i) g_T is of order $(m_u - m_d)$.
ii) Its sign based on the definition of eq.(1) is negative.
iii) g_T/g_A ranges from -0.0205 to -0.00983, i.e., $g_T/g_A \simeq 3 \sim 4(m_u - m_d)/M_N$ which is much smaller than the preliminary experimental value.
iv) By using FESR, we get the analytic formula for g_T:

$$g_T = \frac{25}{32}\frac{\langle\bar{d}d - \bar{u}u\rangle_0}{\langle\bar{u}u\rangle_0} = \frac{25}{32}\frac{C_m(m_u - m_d)}{\langle\bar{u}u\rangle_0}. \tag{75}$$

As mentioned in the introduction, the MIT bag model has been utilized so far to calculate g_T. In this model, we get the following result up to $O(m_u - m_d)$ (see Appendix A for the detailed calculations):

$$g_T = 0.041 M_N(m_u - m_d)R^2, \tag{76}$$

with R being the bag radius. Note that the difference of the bag radius between the proton and the neutron is neglected. By taking $R = 1.085$ fm [31], which reproduces the proton mass, we obtain $g_T = -0.00455$ which is consistent with the result obtained by QSR. Since the obtained result is rather sensitive to the bag radius, one should take this number only qualitatively.

Another effect to g_T, which we must take into account, is the electromagnetic effect. Rough estimate using a hadronic model shows that this effect is smaller than that of the u-d quark mass difference as in the case of the $\rho - \omega$ mixing and the p-n mass difference. Thus our conclusion eq.(74) will not be changed qualitatively by the electromagnetic effect. Nevertheless, more detailed study of QSR with the electromagnetic effect must be done.

In summary we have examined the induced tensor, g_T, in QSR with the external field, and gotten g_T/g_A =-0.0152 $\pm$ 0.0053 which is smaller than preliminary experimntal numbers by one order of magnitude. (current experimental number is ranging from 0.14 ± 0.10 to -0.21 ± 0.14 [15]). However, the experiments and its analyses remain uncertain in order to compare with result obtained in this paper [15,24,32]. Our result should be checked in future beta-decay experiments to understand the G-parity violation.

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APPENDIX A: ESTIMATE OF INDUCED TENSOR IN THE MIT BAG MODEL

We define the matrix element as

$$\langle p(p_2)|J^5_\mu(x)|n(p_1)\rangle = \bar{u}_p(p_2)\left(\frac{i\gamma_5\sigma_{\mu\nu}}{M_n+M_p}g_T q_\nu\right)u_n(p_1) \tag{A1}$$

where $q = p_1 - p_2$.
I)

$$\frac{\partial}{\partial\vec{q}}\langle p|J^5_0(x)|n\rangle|_{\vec{q}=0} = \frac{g_T}{2M_N}\bar{u}_p\vec{\sigma}u_n = \frac{g_T}{2M_N}{}_{s-f}\langle p|\vec{\sigma}\tau^+|n\rangle_{s-f}, \tag{A2}$$

where the subscript denotes a spin-flavor matrix element and, we use

$$\langle p|J^5_\mu(x)|n\rangle = \frac{g_T}{2M_N}\vec{q}\bar{u}_p\begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix}u_n$$

II) The axial current in the MIT bag model is

$$J^5_\mu(x) = \sum_i \bar{q}_i(x)\gamma_\mu\gamma_5\tau^+q_i(x) \tag{A3}$$

From eq.(A2), we obtain

$$\begin{aligned}\frac{\partial}{\partial\vec{q}}\langle p|J^5_0(x)|n\rangle|_{\vec{q}} &= \frac{\partial}{\partial\vec{q}}\int d^3x\ \langle p|J^5_0(x)|n\rangle e^{-i\vec{q}\cdot x}|_{\vec{q}=0}\\ &= -i\int d^3x\ \vec{x}\langle p|\sum_i\bar{q}_i(x)\gamma^0\gamma_5\tau^+q_i(x)|n\rangle.\end{aligned} \tag{A4}$$

Hence,

$$\begin{aligned}&\int_{bag} d^3x\bar{q}_u(x)\gamma^0\gamma^5q_d(x)\vec{x}\\ &= \frac{N_uN_d}{4\pi}\int_{bag} d^3x\hat{x}\ \left\{\sqrt{\frac{E_u+m_u}{E_u}}\sqrt{\frac{E_d-m_d}{E_d}}j_0(x_ur/R)i\vec{\sigma}\cdot\hat{r}j_1(x_dr/R) - (u\leftrightarrow d)\right\}\end{aligned} \tag{A5}$$

where

$$q(x) = \frac{1}{\sqrt{4\pi}}\begin{bmatrix}\sqrt{\frac{E+m}{E}}j_0(xr/R)\\ \sqrt{\frac{E-m}{E}}i\vec{\sigma}\hat{r}j_1(xr/R)\end{bmatrix},\quad E(m,R) = \frac{1}{R}[x^2+(mR)^2]^{1/2}$$

$$N_q^{-2}(x) = R^3j_0^2(x)\frac{2E(E-1/R)+m/R}{E(E-m)},\quad \tan x = \frac{x}{1-mR-[x^2+(mR)^2]^{1/2}},$$

where $j_n(x)$ is spherical Bessel function and R is Bag radius and $r = |\vec{x}|$.
To show the effect of $u-d$ quark mass difference, we expand eq.(A5) up to linear in $m_u - m_d$:

$$
\begin{aligned}
(\text{Eq.A5}) &= \frac{(m_u - m_d)R}{x_0 - 1}\frac{N^2}{4\pi}\int_{bag} d^3x\ r j_0(x_0 r/R) j_1(x_0 r/R) \\
&= -\frac{(m_u - m_d)}{2(x_0 - 1)}\frac{N^2}{4\pi}\int_{bag} d^3x\ r^2 (j_0^2(x_0 r/R) + j_1^2(x_0 r/R))
\end{aligned} \tag{A6}
$$

where

$$
\tan x_0 = \frac{x_0}{1 - x_0}, \quad E(m, R) = \frac{x_0}{R} + \frac{m}{2(x_0 - 1)} + O(m^2). \tag{A7}
$$

Thus, we get

$$
\begin{aligned}
& \frac{\partial}{\partial \vec{q}} \langle p | J_0^5(x) | n \rangle |_{\vec{q}} \\
& \times \frac{1}{3}{}_{s-f}\langle p | \sum_i \vec{\sigma}_i \cdot \tau_i^+ | n \rangle_{s-f} \frac{(m_u - m_d)N^2}{4\pi(x_0 - 1)} \\
& = \left(R \int d^3x\ r j_0(x_0 r/R) j_1(x_0 r/R) - \frac{1}{2} \int d^3x\ r^2\ (j_0^2(x_0 r/R) + j_1^2(x_0 r/R) \right).
\end{aligned} \tag{A8}
$$

Equating eq.(A2) to eq.(A8), we arrive at

$$
\frac{g_T}{2M_N} = -(m_u - m_d)\frac{5R^2}{36 x_0 (x_0^2 - 1)^2}\left[\frac{1}{x_0} - \frac{17}{6} + \frac{8}{3}x_0 - \frac{2}{3}x_0^2\right], \tag{A9}
$$

where $x_0 = 2.04$ and we have used

$$
{}_{s-f}\langle p | \sum_i \vec{\sigma}_i \cdot \tau_i^+ | n \rangle_{s-f} = \frac{5}{3}{}_{s-f}\langle p | \vec{\sigma} \cdot \tau^+ | n \rangle_{s-f}. \tag{A10}
$$

Figure Captions

Fig.1

A schematic illustration that neuton absorbes W^+ boson and turns into proton.

Fig.2

OPE for $\Pi_E(p)$, where for the chiral odd structures $\Pi_E(p)$ is expanded up to dimension 8 with $m_u = m_d$, while for the chiral even strutures $\Pi_E(p)$ is expanded up to dimension 5, and up to linear in $(m_u - m_d)$. Dashed lines denotes the external field, wavy lines denote gluon lines,and broken lines denote the quark/gluon condensate.

Fig.3

OPE up to dimension 5 for $\chi(p)$ sum rules. Wavy lines denote gluon lines and broken lines denote the quark/gluon condensate.

Fig.4

OPE up to dimension 7 for $\kappa(p)$ and $\xi(p)$ sum rules. Wavy lines denote gluon lines and broken lines denote the quark/gluon condensate.

Fig.5 (a),(b)

g_T^{even} and g_T^{odd} with the optimal threshold S_T as a function of the Borel mass squared M^2. S_T is also shown in GeV^2 unit. (a)((b)) corresponds to $g_T^{even}(g_T^{odd})$. The solid (dashed) line corresponds to $C_m(\mathrm{GeV}^2) = -0.0306$ (-0.0223),

Fig.6

g_T/g_A with the optimal threshold S_T as a function of the Borel mass squared M^2. S_T is also shown in GeV^2 unit. The solid (dashed) line corresponds to $C_m(\mathrm{GeV}^2) = -0.0306(-0.0223)$.

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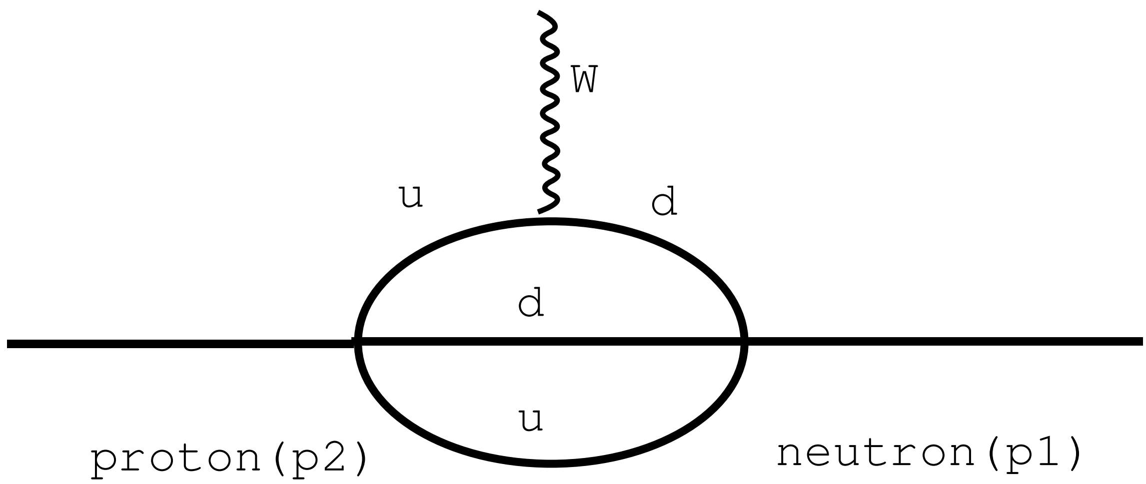


Fig.1

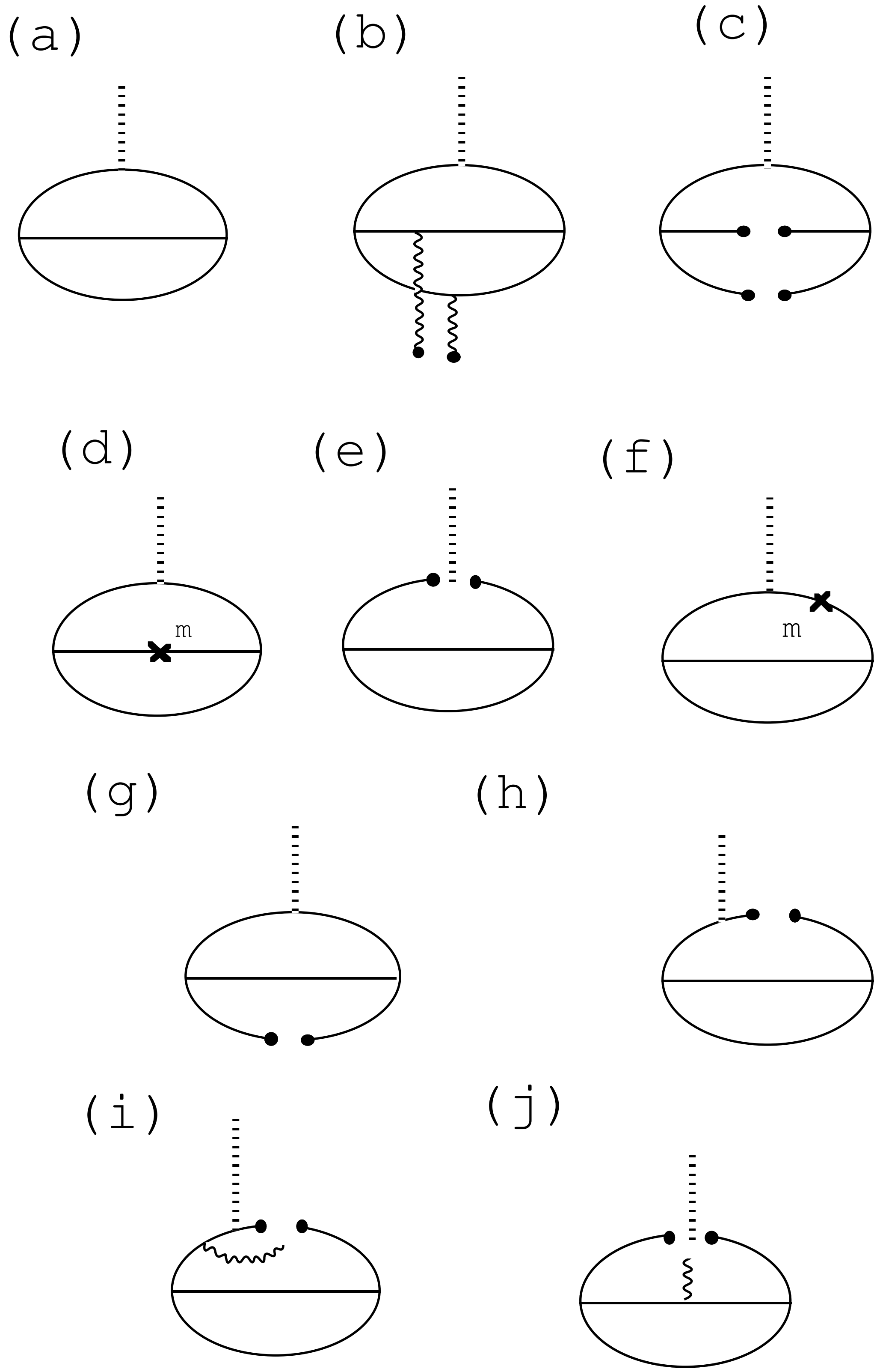


Fig. 2

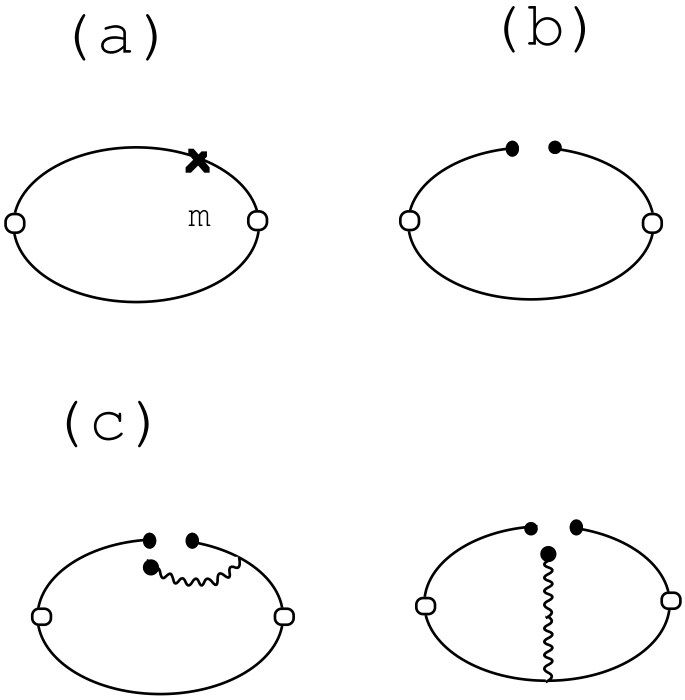
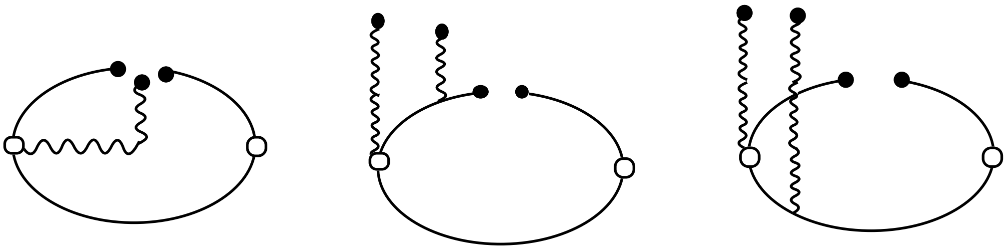


Fig.3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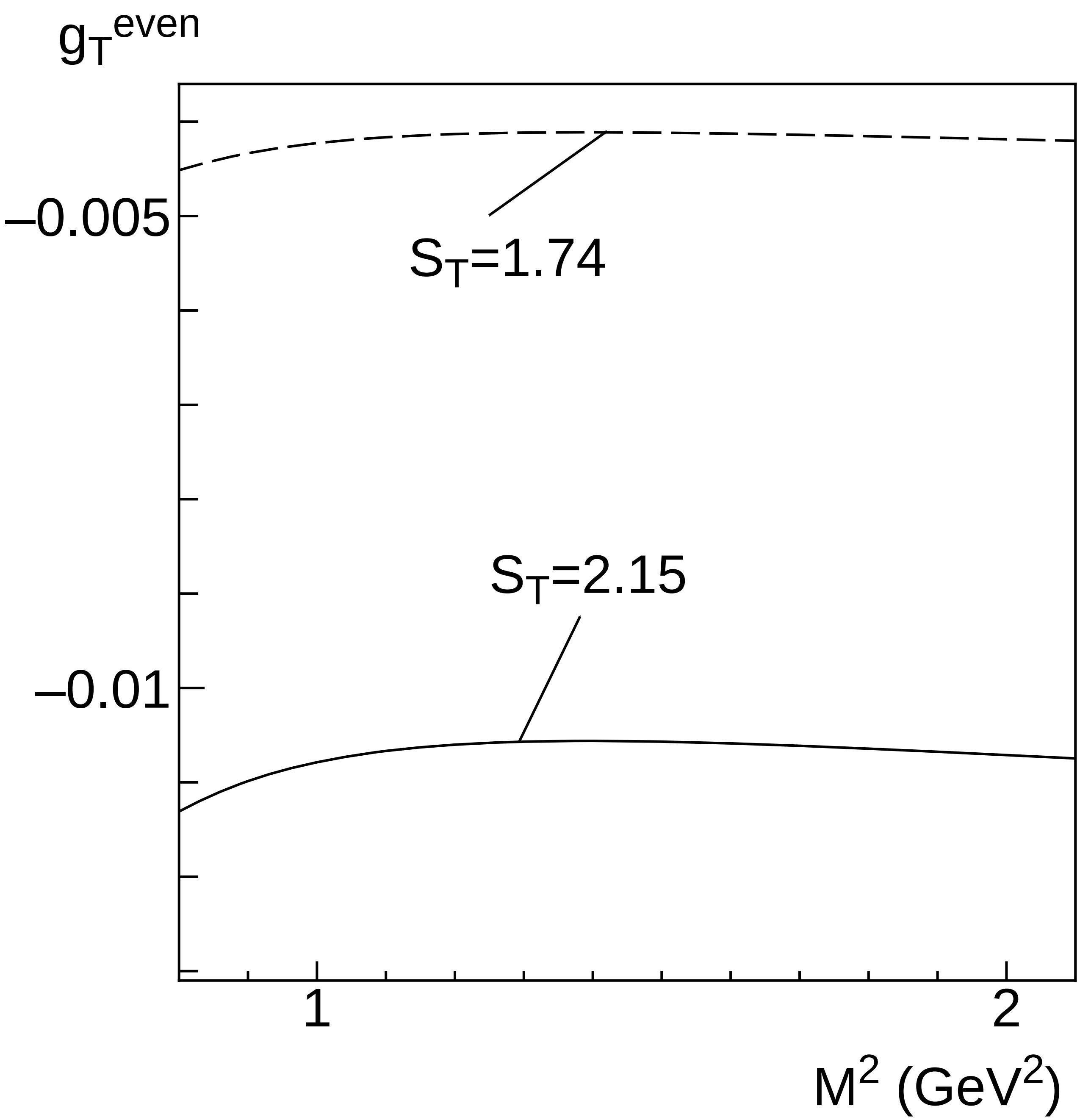
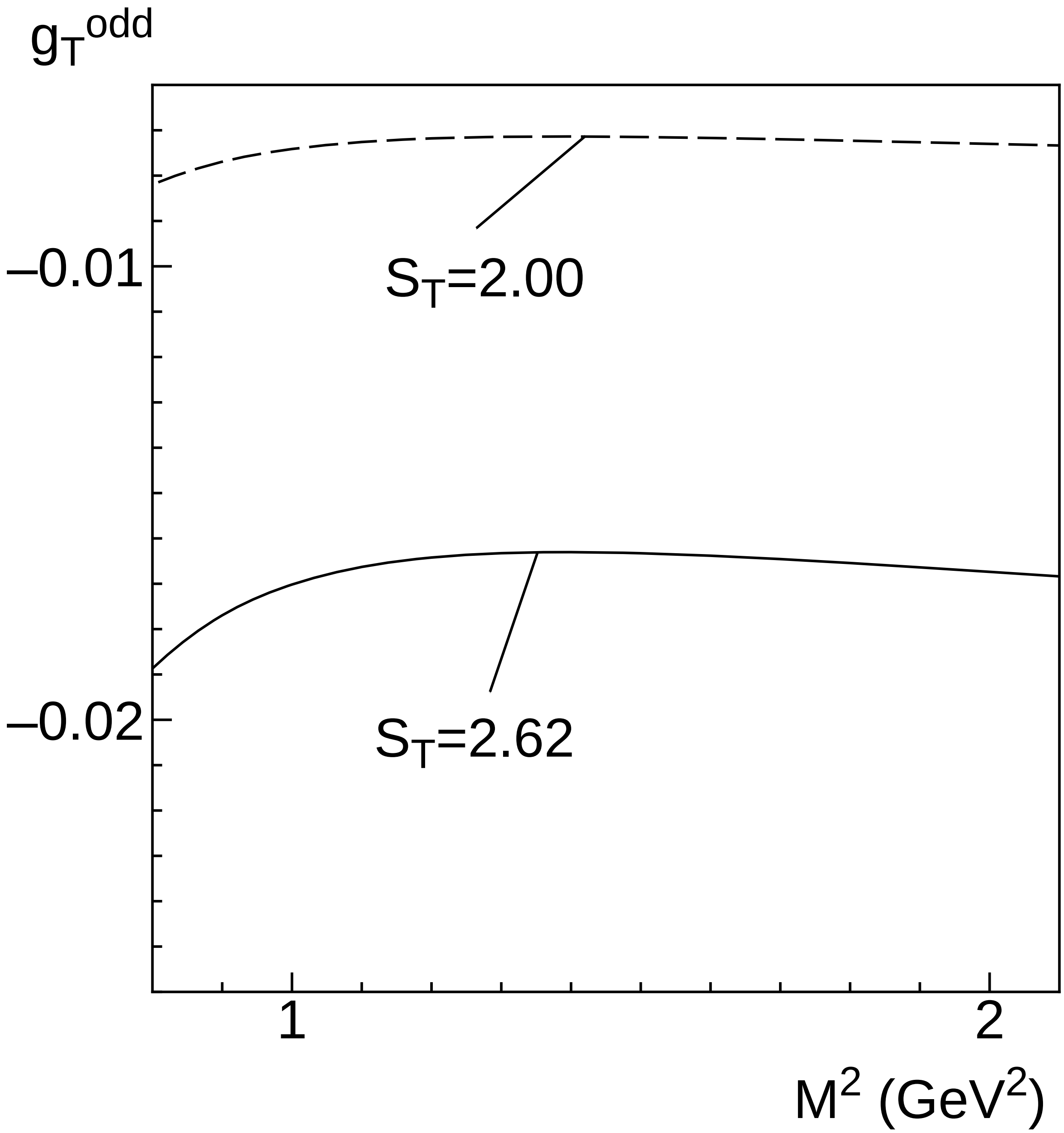


Fig. 5(a)



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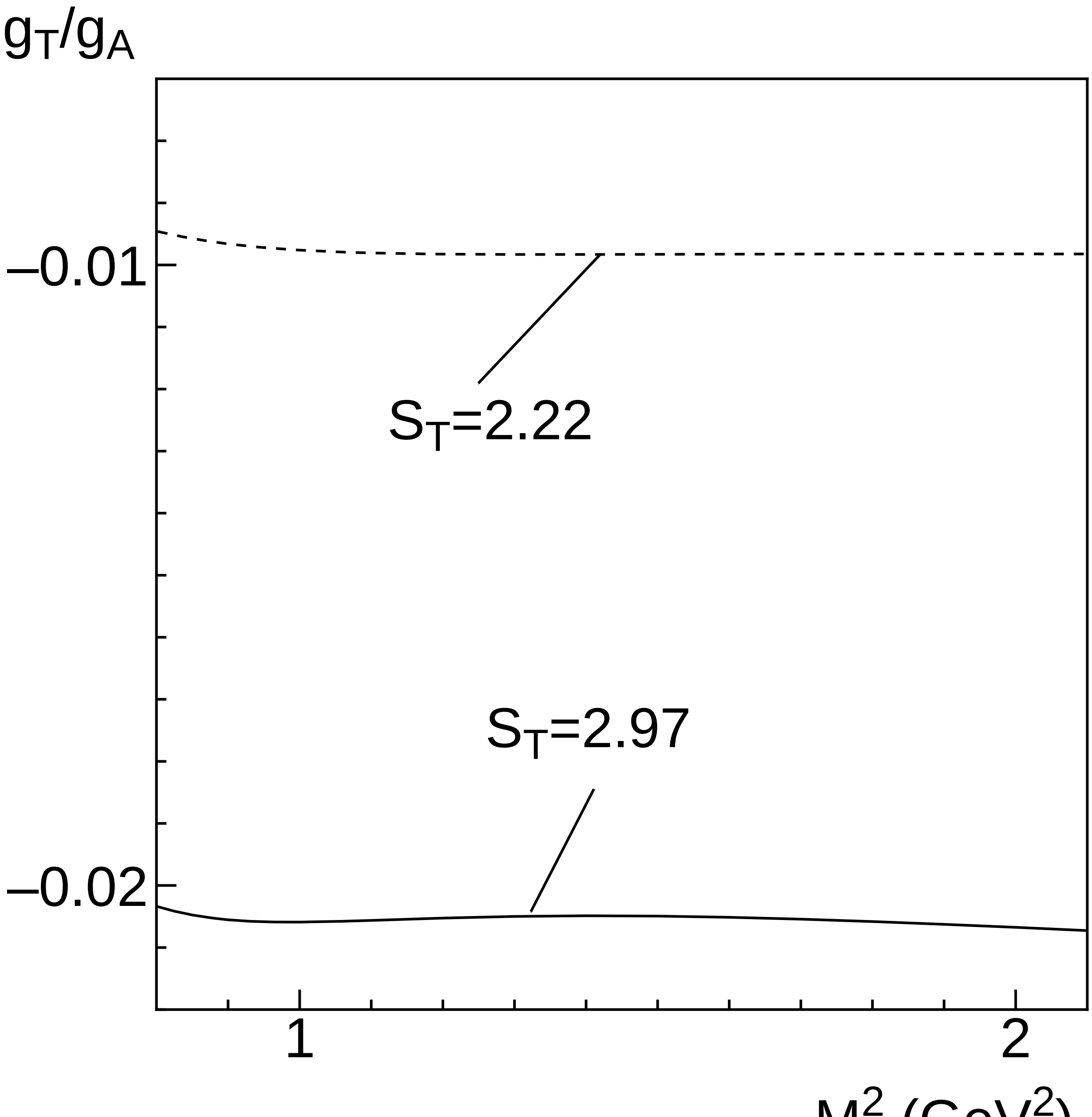

Fig.6